Potentially $K_m - G$-graphical Sequences: A Survey *

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Abstract

The set of all non-increasing nonnegative integers sequence $\pi = (d(v_1), d(v_2), ..., d(v_n))$ is denoted by NS_n. A sequence $\pi \in NS_n$ is said to be graphic if it is the degree sequence of a simple graph G on n vertices, and such a graph G is called a realization of π. The set of all graphic sequences in NS_n is denoted by GS_n. A graphical sequence π is potentially H-graphical if there is a realization of π containing H as a subgraph, while π is forcibly H-graphical if every realization of π contains H as a subgraph. Let K_k denote a complete graph on k vertices. Let $K_m - H$ be the graph obtained from K_m by removing the edges set $E(H)$ of the graph H (H is a subgraph of K_m). This paper summarizes briefly some recent results on potentially $K_m - G$-graphic sequences and give a useful classification for determining $\sigma(H, n)$.
Key words: graph; degree sequence; potentially $K_m - G$-graphic sequences
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1 Introduction

The set of all non-increasing nonnegative integer sequence $\pi = (d(v_1), d(v_2), ..., d(v_n))$ is denoted by NS_n. A sequence $\pi \in NS_n$ is said to be graphic

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if it is the degree sequence of a simple graph G on n vertices, and such a graph G is called a realization of π. The set of all graphic sequences in NS_n is denoted by GS_n. A graphical sequence π is potentially H-graphical if there is a realization of π containing H as a subgraph, while π is forcibly H-graphical if every realization of π contains H as a subgraph. If π has a realization in which the $r+1$ vertices of largest degree induce a clique, then π is said to be potentially A_{r+1}-graphic. Let $\sigma(\pi) = d(v_1)+d(v_2)+...+d(v_n)$, and $[x]$ denote the largest integer less than or equal to x. We denote $G + H$ as the graph with $V(G + H) = V(G) \bigcup V(H)$ and $E(G + H) = E(G) \bigcup E(H) \bigcup \{xy : x \in V(G), y \in V(H)\}$. Let K_k, C_k, T_k, and P_k denote a complete graph on k vertices, a cycle on k vertices, a tree on $k+1$ vertices, and a path on $k+1$ vertices, respectively. Let F_k denote the friendship graph on $2k + 1$ vertices, that is, the graph of k triangles intersecting in a single vertex. For $0 \le r \le t$, denote the generalized friendship graph on $kt-kr+r$ vertices by $F_{t,r,k}$, where $F_{t,r,k}$ is the graph of k copies of K_t meeting in a common r set. We use the symbol Z_4 to denote $K_4 - P_2$. Let $K_m - H$ be the graph obtained from K_m by removing the edges set $E(H)$ of the graph H (H is a subgraph of K_m).

Given a graph H, what is the maximum number of edges of a graph with n vertices not containing H as a subgraph? This number is denoted $ex(n, H)$, and is known as the Turán number. In terms of graphic sequences, the number $2ex(n, H) + 2$ is the minimum even integer l such that every n-term graphical sequence π with $\sigma(\pi) \ge l$ is forcibly H-graphical. Erdös, Jacobson and Lehel [13] first consider the following variant: determine the minimum even integer l such that every n-term graphical sequence π with $\sigma(\pi) \ge l$ is potentially H-graphical. We denote this minimum l by $\sigma(H, n)$. Erdös, Jacobson and Lehel [13] showed that $\sigma(K_k, n) \ge (k - 2)(2n - k + 1) + 2$ and conjectured that equality holds. They proved that if π does not contain zero terms, this conjecture is true for $k = 3$, $n \ge 6$. The conjecture is confirmed in [19] and [43-46]. Li et al. [46] and Mubayi [55] also independently determined the values $\sigma(K_r, 2k)$ for any $k \ge 3$ and $n \ge k$. Li and Yin [51] further determined $\sigma(K_r, n)$ for $r \ge 7$ and $n \ge 2r+1$. The problem of determining $\sigma(K_r, n)$ is completely solved.

Gould, Jacobson and Lehel[19] also proved that $\sigma(pK_2, n) = (p-1)(2n-p)+2$ for $p \ge 2$; $\sigma(C_4, n) = 2[\frac{3n-1}{2}]$ for $n \ge 4$. Lai[29] gave a lower bound of $\sigma(C_k, n)$ and proved that $\sigma(C_5, n) = 4n-4$ for $n \ge 5$ and $\sigma(C_6, n) = 4n-2$ for $n \ge 7$. Lai [32] proved that $\sigma(C_{2m+1}, n) = m(2n - m - 1) + 2$, for $m \ge 2, n \ge 3m$; $\sigma(C_{2m+2}, n) = m(2n - m - 1) + 4$, for $m \ge 2, n \ge 5m - 2$. Li and Luo[41] gave a lower bound of $\sigma({}_3C_l, n)$ and determined $\sigma({}_3C_l, n)$, $4 \le l \le 6, n \ge l$. Li, Luo and Liu [42] determined $\sigma({}_3C_l, n)$ for $3 \le l \le 8$ and $n \ge l$. and $\sigma({}_3C_9, n)$ for $n \ge 12$. Li and Yin [48] determined $\sigma({}_3C_l, n)$ for n sufficiently large. Yin, Li and Chen[68] determined $\sigma({}_kC_l, n)$, $l \ge 7, 3 \le k \le l$. Chen and Yin[9] determined the values

$\sigma(W_5, n)$ for $n \geq 11$ where W_r is a wheel graph on r vertices. For $r \times s$ complete bipartite graph $K_{r,s}$, Gould, Jacobson and Lehel[19] determined $\sigma(K_{2,2}, n)$. Yin et al. [63,65,69,70] determined $\sigma(K_{r,s}, n)$ for $s \geq r \geq 2$ and sufficiently large n. For $r \times s \times t$ complete 3-partite graph $K_{r,s,t}$, Erdös, Jacobson and Lehel[13] determined $\sigma(K_{1,1,1}, n)$. Lai[30] determined $\sigma(K_{1,1,2}, n)$. Yin[58] and Lai[34] independently determined $\sigma(K_{1,1,3}, n)$. Chen[7] determined $\sigma(K_{1,1,t}, n)$ for $t \geq 3$, $n \geq 2[\frac{(t+5)^2}{4}] + 3$. Chen[5] determined $\sigma(K_{1,2,2}, n)$ for $5 \leq n \leq 8$ and $\sigma(K_{2,2,2}, n)$ for $n \geq 6$. Let K_s^t denote the complete t partite graph such that each partite set has exactly s vertices. Guantao Chen, Michael Ferrara, Ronald J.Gould, John R. Schmitt[11] showed that $\sigma(K_s^t, n) = \pi(K_{(t-2)s} + K_{s,s}, n)$ and obtained the exact value of $\sigma(K_j + K_{s,s}, n)$ for n sufficiently large. Consequently, they obtained the exact value of $\sigma(K_s^t, n)$ for n sufficiently large. For $n \geq 5$, Ferrara, Jacobson and Schmitt[17] determined $\sigma(F_k, n)$ where F_k denotes the graph of k triangles intersecting at exactly one common vertex. In[16], Ferrara, Gould and Schmitt determined a lower bound for $\sigma(K_s^t, n)$ where K_s^t denotes the complete multipartite graph with t partite sets each of size s, and proved equality in the case $s = 2$. They also provided a graph theoretic proof of the value of $\sigma(K_t, n)$. Michael J. Ferrara[15] determined $\sigma(H, n)$ for the graph $H = K_{m_1} \cup K_{m_2} \cup \cdots \cup K_{m_k}$ where n is sufficiently large integer. Ferrara, M., Jacobson, M., Schmitt, J. and Siggers M.[18] determined $\sigma(K_{s,t}, m, n)$, $\sigma(P_t, m, n)$ and $\sigma(C_{2t}, m, n)$ where $\sigma(H, m, n)$ is the minimum integer k such that every bigraphic pair $S = (A, B)$ with $|A| = m$, $|B| = n$ and $\sigma(S) \geq k$ is potentially H-bigraphic. For an arbitrarily chosen H, Schmitt, J.R. and Ferrara, M.[56] gave a good lower bound of $\sigma(H, n)$. Yin and Li[67] determined $\sigma(K_{r_1,r_2,\cdots,r_l,r,s}, n)$ for sufficiently large n. Moreover, Yin, Chen and Schmitt[62] determined $\sigma(F_{t,r,k}, n)$ for $k \geq 2, t \geq 3, 1 \leq r \leq t-2$ and sufficiently large, where $F_{t,r,k}$ denotes the graph of k copies of K_t meeting in a common r set. Gupta, Joshi and Tripathi [20] gave a necessary and sufficient condition for the existence of a tree of order n with a given degree set. Yin [59] gave a new necessary and sufficient condition for π to be potentially K_{r+1}- graphic. Jiong-sheng Li and Jianhua Yin [50] gave a survey on graphical sequences.

2 Potentially $K_m - G$-graphical Sequences

Let H be a graph with m vertices, then $H = K_m - (K_m - H)$. Let $G = K_m - H$, then $\sigma(H, n) = \sigma(K_m - G, n)$. If the Problem 1 - 5 in the Open Problems section be solved, then the problem of determining $\sigma(H, n)$ is completely solved. We think the Problem 3, 4 is a useful classification for determining $\sigma(K_m - G, n)$.

Gould, Jacobson and Lehel[19] pointed out that it would be nice to see

where in the range for $3n-2$ to $4n-4$, the value $\sigma(K_4-e,n)$ lies. Later, Lai[30] proved that

Theorem 1. For $n=4,5$ and $n\geq 7$

$$\sigma(K_4-e,n)=\begin{cases} 3n-1 & \text{if } n \text{ is odd} \\ 3n-2 & \text{if } n \text{ is even.} \end{cases}$$

For $n=6$, S is a 6-term graphical sequence with $\sigma(S)\geq 16$, then either there is a realization of S containing K_4-e or $S=(3^6)$. (Thus $\sigma(K_4-e,6)=20$).

Huang[26] gave a lower bound of $\sigma(K_m-e,n)$. Yin, Li and Mao[71] and Huang[27] independently determined the values $\sigma(K_5-e,n)$ as following.

Theorem 2. If $n\geq 5$, then

$$\sigma(K_5-e,n)=\begin{cases} 5n-7, & \text{if } n \text{ is odd} \\ 5n-6, & \text{if } n \text{ is even} \end{cases}$$

Lai[35-36] determined $\sigma(K_5-C_4,n),\sigma(K_5-P_3,n)$ and $\sigma(K_5-P_4,n)$.

Theorem 3. For $n\geq 5$, $\sigma(K_5-C_4,n)=\sigma(K_5-P_3,n)=\sigma(K_5-P_4,n)=4n-4$.

Yin and Li[66] gave a good method of determining the values $\sigma(K_{r+1}-e,n)$ (In fact, Yin and Li[66] also determining the values $\sigma(K_{r+1}-ke,n)$ for $r\geq 2$ and $n\geq 3r^2-r-1$).

Theorem 4. Let $n\geq r+1$ and $\pi=(d_1,d_2,\cdots,d_n)\in GS_n$ with $d_{r+1}\geq r$. If $d_i\geq 2r-i$ for $i=1,2,\cdots,r-1$, then π is potentially A_{r+1}-graphic.

Theorem 5. Let $n\geq 2r+2$ and $\pi=(d_1,d_2,\cdots,d_n)\in GS_n$ with $d_{r+1}\geq r$. If $d_{2r+2}\geq r-1$, then π is potentially A_{r+1}-graphic.

Theorem 6. Let $n\geq r+1$ and $\pi=(d_1,d_2,\cdots,d_n)\in GS_n$ with $d_{r+1}\geq r-1$. If $d_i\geq 2r-i$ for $i=1,2,\cdots,r-1$, then π is potentially $K_{r+1}-e$-graphic.

Theorem 7. Let $n\geq 2r+2$ and $\pi=(d_1,d_2,\cdots,d_n)\in GS_n$ with $d_{r-1}\geq r$. If $d_{2r+2}\geq r-1$, then π is potentially $K_{r+1}-e$ -graphic.

Theorem 8. If $r\geq 2$ and $n\geq 3r^2-r-1$, then

$$\sigma(K_{r+1}-ke,n)=\begin{cases} (r-1)(2n-r)-(n-r)+1, \\ \text{ if } n-r \text{ is odd} \\ (r-1)(2n-r)-(n-r)+2, \\ \text{if } n-r \text{ is even} \end{cases}$$

After reading[66], Yin[72] determined the values $\sigma(K_{r+1}-K_3,n)$ for $r\geq 3, n\geq 3r+5$.

Theorem 9. If $r\geq 3$ and $n\geq 3r+5$, then $\sigma(K_{r+1}-K_3,n)=(r-1)(2n-r)-2(n-r)+2$.

Determining $\sigma(K_{r+1} - H, n)$, where H is a tree on 4 vertices is more useful than a cycle on 4 vertices (for example, $C_4 \not\subset C_i$, but $P_3 \subset C_i$ for $i \geq 5$). So, after reading[66] and [72], Lai and Hu[38] determined $\sigma(K_{r+1} - H, n)$ for $n \geq 4r+10, r \geq 3, r+1 \geq k \geq 4$ and H be a graph on k vertices which containing a tree on 4 vertices but not containing a cycle on 3 vertices and $\sigma(K_{r+1} - P_2, n)$ for $n \geq 4r+8, r \geq 3$.

Theorem 10. If $r \geq 3$ and $n \geq 4r+8$, then $\sigma(K_{r+1} - P_2, n) = (r-1)(2n-r) - 2(n-r) + 2$.

Theorem 11. If $r \geq 3, r+1 \geq k \geq 4$ and $n \geq 4r+10$, then $\sigma(K_{r+1} - H, n) = (r-1)(2n-r) - 2(n-r)$, where H is a graph on k vertices which contains a tree on 4 vertices but not contains a cycle on 3 vertices.

There are a number of graphs on k vertices which containing a tree on 4 vertices but not containing a cycle on 3 vertices (for example, the cycle on k vertices, the tree on k vertices, and the complete 2-partite graph on k vertices, etc).

Lai and Sun[39] determined $\sigma(K_{r+1} - (kP_2 \bigcup tK_2), n)$ for $n \geq 4r+10, r+1 \geq 3k+2t, k+t \geq 2, k \geq 1, t \geq 0$.

Theorem 12. If $n \geq 4r+10, r+1 \geq 3k+2t, k+t \geq 2, k \geq 1, t \geq 0$, then $\sigma(K_{r+1} - (kP_2 \bigcup tK_2), n) = (r-1)(2n-r) - 2(n-r)$.

To now, the problem of determining $\sigma(K_{r+1} - H, n)$ for H not containing a cycle on 3 vertices and sufficiently large n has been solved.

Lai[37] determined $\sigma(K_{r+1} - Z, n)$ for $n \geq 5r+19, r+1 \geq k \geq 5, j \geq 5$ and Z is a graph on k vertices and j edges which contains a graph Z_4 but not contain a cycle on 4 vertices. In the same paper, the author also determined the values of $\sigma(K_{r+1} - Z_4, n)$, $\sigma(K_{r+1} - (K_4 - e), n)$, $\sigma(K_{r+1} - K_4, n)$ for $n \geq 5r+16, r \geq 4$.

Theorem 13. If $r \geq 4$ and $n \geq 5r+16$, then

$$\sigma(K_{r+1} - K_4, n) = \sigma(K_{r+1} - (K_4 - e), n) =$$

$$\sigma(K_{r+1} - Z_4, n) = \begin{cases} (r-1)(2n-r) - 3(n-r) + 1, \\ \text{ if } n-r \text{ is odd} \\ (r-1)(2n-r) - 3(n-r) + 2, \\ \text{if } n-r \text{ is even} \end{cases}$$

Theorem 14. If $n \geq 5r+19, r+1 \geq k \geq 5$, and $j \geq 5$, then

$$\sigma(K_{r+1} - Z, n) = \begin{cases} (r-1)(2n-r) - 3(n-r) - 1, \\ \text{ if } n-r \text{ is odd} \\ (r-1)(2n-r) - 3(n-r) - 2, \\ \text{if } n-r \text{ is even} \end{cases}$$

where Z is a graph on k vertices and j edges which contains a graph Z_4 but not contains a cycle on 4 vertices.

There are a number of graphs on k vertices and j edges which contains a graph Z_4 but not contains a cycle on 4 vertices. (for example, the graph obtained by $C_3, C_{i_1}, C_{i_2}, \cdots, C_{i_p}$ intersecting in a single vertex ($i_j \neq 4, j = 1, 2, 3, \cdots, p$) (if $i_j = 3, j = 1, 2, 3, \cdots, p$, then this graph is the friendship graph F_{p+1},), the graph obtained by $C_3, P_{i_1}, P_{i_2}, \cdots, P_{i_p}$ intersecting in a single vertex, ($i_1 \geq 1$), the graph obtained by $C_3, P_{i_1}, C_{i_2}, \cdots, C_{i_p}$ ($i_j \neq 4, j = 2, 3, \cdots, p, i_1 \geq 1$) intersecting in a single vertex, etc)

Lai and Yan[40] proved that

Theorem 15. If $n \geq 5r + 18, r + 1 \geq k \geq 7$, and $j \geq 6$, then

$$\sigma(K_{r+1} - U, n) = \begin{cases} (r-1)(2n-r) - 3(n-r) - 1, \\ \text{ if } n-r \text{ is odd} \\ (r-1)(2n-r) - 3(n-r), \\ \text{if } n-r \text{ is even} \end{cases}$$

where U is a graph on k vertices and j edges which contains a graph $(K_3 \bigcup P_3)$ but not contains a cycle on 4 vertices and not contains Z_4.

There are a number of graphs on k vertices and j edges which contains a graph $(K_3 \bigcup P_3)$ but not contains a cycle on 4 vertices and not contains Z_4. (for example, $C_3 \bigcup C_{i_1} \bigcup C_{i_2} \bigcup \cdots \bigcup C_{i_p}$ ($i_j \neq 4, j = 2, 3, \cdots, p, i_1 \geq 5$), $C_3 \bigcup P_{i_1} \bigcup P_{i_2} \bigcup \cdots \bigcup P_{i_p}$ ($i_1 \geq 3$), $C_3 \bigcup P_{i_1} \bigcup C_{i_2} \bigcup \cdots \bigcup C_{i_p}$ ($i_j \neq 4, j = 2, 3, \cdots, p, i_1 \geq 3$), etc)

A harder question is to characterize the potentially H-graphic sequences without zero terms. Luo [53] characterized the potentially C_k-graphic sequences for each $k = 3, 4, 5$.

Theorem 16. Let $\pi = (d_1, d_2, \cdots, d_n)$ be a graphic sequence with $n \geq 3$. Then π is potentially C_3-graphic if and only if $d_3 \geq 2$ except for 2 case: $\pi = (2^4)$ and $\pi = (2^5)$.

Theorem 17. Let $\pi = (d_1, d_2, \cdots, d_n)$ be a graphic sequence with $n \geq 4$. Then π is potentially C_4-graphic if and only if the following conditions hold:

(1) $d_4 \geq 2$. (2) $d_1 = n - 1$ implies $d_2 \geq 3$

(3) If $n = 5, 6$, then $\pi \neq (2^n)$.

Theorem 18. Let $\pi = (d_1, d_2, \cdots, d_n)$ be a graphic sequence with $n \geq 5$. Then π is potentially C_5-graphic if and only if the following conditions hold:

(1) $d_5 \geq 2$. (2) For $i = 1, 2, d_1 = n - i$ implies $d_{4-i} \geq 3$

(3) If $\pi = (d_1, d_2, 2^k, 1^{n-k-2})$, then $d_1 + d_2 \leq n + k - 2$.

Chen [2] characterized the potentially C_k-graphic sequences for each $k = 6$.

Theorem 19. Let $\pi = (d_1, d_2, \cdots, d_n)$ be a graphic sequence with $n \geq 6$. Then π is potentially C_6-graphic if and only if the following conditions hold:

(1) $d_6 \geq 2$.

(2) If $n = 7, 8$, then $\pi \neq (2^n)$.

(3) For $i = 1, 2, 3, d_1 = n - i$ implies $d_{5-i} \geq 3$

(4) If $\pi = (d_1, d_2, 2^k, 1^{n-k-2})$, then $d_1 + d_2 \leq n + k - 2$; if $\pi = (d_1, d_2, 3, 2^k, 1^{n-k-3})$, then $d_1 + d_2 \leq n + k$; if $\pi = (d_1, d_2, 3, 3, 2^k, 1^{n-k-4})$, then $d_1 + d_2 \leq n + k + 2$.

Yin, Chen and Chen [60] characterized the potentially ${}_kC_l$-graphic sequences for each $k = 3, 4 \leq l \leq 5$ and $k = 4, l = 5$.

Theorem 20. Let $\pi = (d_1, d_2, \cdots, d_n) \in GS_n$ be a potentially C_4-graphic sequence. Then π is potentially ${}_3C_4$-graphic if and only if π satisfies one of the following conditions:

(1) $d_2 \geq 3$ and $\pi \neq (3^2, 2^4)$;

(2) $\pi = (d_1, 2^k, 1^{n-k-1})$ with $2 \leq d_1 \leq 3$ and $k \geq 6$, and $\pi \neq (2^8)$ and (2^9);

(3) $\pi = (d_1, 2^k, 1^{n-k-1})$ with $4 \leq d_1 \leq n-2$ and $k \geq 5$, and $\pi \neq (4, 2^6)$ and $(4, 2^7)$.

Theorem 21. Let $\pi = (d_1, d_2, \cdots, d_n) \in GS_n$ be a potentially C_5-graphic sequence. Then π is potentially ${}_3C_5$-graphic if and only if π satisfies one of the following conditions:

(1) $d_2 \geq 3$ and $\pi \neq (3^2, 2^4)$ and $(3^2, 2^5)$;

(2) $\pi = (d_1, 2^k, 1^{n-k-1})$ with $2 \leq d_1 \leq 3$ and $k \geq 11$, and $\pi \neq (2^{13})$ and (2^{14});

(3) $\pi = (d_1, 2^k, 1^{n-k-1})$ with $4 \leq d_1 \leq 5$ and $k \geq 10$, and $\pi \neq (4, 2^{11})$ and $(4, 2^{12})$.

(4) $\pi = (d_1, 2^k, 1^{n-k-1})$ with $6 \leq d_1 \leq n-4$ and $k \geq 9$, and $\pi \neq (4, 2^{10})$ and $(4, 2^{11})$.

Theorem 22. Let $\pi = (d_1, d_2, \cdots, d_n) \in GS_n$ be a potentially C_5-graphic sequence. Then π is potentially ${}_4C_5$-graphic if and only if π satisfies one of the following conditions:

(1) $d_2 \geq 3$;

(2) $\pi = (d_1, 2^k, 1^{n-k-1})$ with $2 \leq d_1 \leq 3$ and $k \geq 8$, and $\pi \neq (2^{10})$ and (2^{11});

(3) $\pi = (d_1, 2^k, 1^{n-k-1})$ with $4 \leq d_1 \leq n-4$ and $k \geq 7$, and $\pi \neq (4, 2^8)$ and $(4, 2^9)$.

Chen, Yin and Fan [10] characterized the potentially ${}_kC_l$-graphic sequences for each $3 \leq k \leq 5, l = 6$.

Theorem 23. Let $\pi = (d_1, d_2, \cdots, d_n) \in GS_n$, $n \geq 6$, and $\pi \neq (3^2, 2^{10})$,(2^{19}), (2^{20}), $(4, 2^{17})$, $(4, 2^{18})$, $(6, 2^{16})$, $(6, 2^{17})$, $(8, 2^{15})$, $(8, 2^{16})$, Then π is potentially ${}_3C_6$-graphic if and only if π be a potentially C_6-graphic sequence, and π satisfies one of the following conditions:

(1) $d_3 \geq 3$, and if $d_1 = d_3 = 3, d_4 = 2$, then $d_{10} = 2$;

(2) $d_2 \geq 4, d_3 = 2, d_7 = 2$;

(3) $d_2 = 3, d_3 = 2$, and if $4 \geq d_1 \geq 3$, then $d_{10} = 2$, and if $n-4 \geq d_1 \geq 5$, then $d_9 = 2$.

(4) $d_2 = 2$, and if $3 \geq d_1 \geq 2$, then $d_{18} = 2$, and if $5 \geq d_1 \geq 4$, then $d_{17} = 2$, and if $7 \geq d_1 \geq 6$, then $d_{16} = 2$, and if $n - 7 \geq d_1 \geq 8$, then $d_{15} = 2$.

Theorem 24. Let $\pi = (d_1, d_2, \cdots, d_n) \in GS_n$, $n \geq 6$, and $\pi \neq (2^{16})$, (2^{17}), $(4, 2^{14})$, $(4, 2^{15})$, $(6, 2^{13})$, $(6, 2^{14})$, Then π is potentially ${}_4C_6$-graphic if and only if π be a potentially C_6-graphic sequence, and π satisfies one of the following conditions:

(1) $d_3 \geq 3$, and if $d_1 = d_3 = 3, d_4 = 2$, then $d_{10} = 2$;

(2) $d_2 \geq 4, d_3 = 2, d_7 = 2$;

(3) $d_2 = 3, d_3 = 2$, and if $4 \geq d_1 \geq 3$, then $d_{10} = 2$, and if $n-4 \geq d_1 \geq 5$, then $d_9 = 2$.

(4) $d_2 = 2$, and if $3 \geq d_1 \geq 2$, then $d_{15} = 2$, and if $5 \geq d_1 \geq 4$, then $d_{14} = 2$, and if $n - 7 \geq d_1 \geq 6$, then $d_{13} = 2$.

Theorem 25. Let $\pi = (d_1, d_2, \cdots, d_n) \in GS_n$, $n \geq 6$, and $\pi \neq (2^{12})$, (2^{13}), $(4, 2^{10})$, $(4, 2^{11})$, Then π is potentially ${}_5C_6$-graphic if and only if π be a potentially C_6-graphic sequence, and π satisfies one of the following conditions:

(1) $d_2 \geq 3$;

(2) $3 \geq d_1 \geq 2, d_2 = 2, d_11 = 2$;

(3) $n - 6 \geq d_1 \geq 4, d_2 = 2, d_{10} = 2$.

Luo and Warner [54] characterized the potentially K_4-graphic sequences.

Theorem 26. Let $\pi = (d_1, d_2, \cdots, d_n)$ be a graphic sequence without zero terms and with $d_4 \geq 3$ and $n \geq 4$. Then π is potentially K_4-graphic if and only if $d_4 \geq 3$ and $\pi \neq (n-1, 3^s, !^{n-s-1})$ for each $s = 4, 5$ except the following sequences:

$n = 5 : (4, 3^4)$, $(3^4, 2)$;

$n = 6 : (4^6)$, $(4^2, 3^4)$, $(4, 3^4, 2)$, (3^6), $(3^5, 1)$, $(3^4, 2^2)$;

$n = 7 : (4^7)$, $(4^3, 3^4)$, $(4, 3^6)$, $(4, 3^5, 1)$, $(3^6, 2)$, $3^5, 2, 1)$;

$n = 8 : (3^7, 1)$, $(3^6, 1^2)$.

Eschen and Niu [14] and Lai[31] independently characterized the potentially $K_4 - e$-graphic sequences.

Theorem 27. Let $\pi = (d_1, d_2, \cdots, d_n)$ be a graphic sequence with $n \geq 4$. Then π is potentially $(K_4 - e)$-graphic if and only if the following conditions hold:

(1) $d_2 \geq 3$. (2) $d_4 \geq 2$

(3) If $n = 5, 6$, then $\pi \neq (3^2, 2^{n-2})$ and $\pi \neq (3^6)$.

Yin and Yin [73] characterized the potentially $K_5 - e$ and K_6-graphic sequences.

Theorem 28 Let $n \geq 5$ and $\pi = (d_1, d_2, \cdots, d_n) \in NS_n$ be a positive graphic sequence with $d_3 \geq 4$ and $d_5 \geq 3$. Then π is potentially

K_5-e-graphic if and only if π is not one of the following sequences: $(n-1,4^6,1^{n-7})$, $(n-1,4^2,3^4,1^{n-7})$, $(n-1,4^2,3^3,1^{n-6})$;

$n=6$: $(4^6),(4^4,3^2),(4^3,3^2,2)$;

$n=7$: $(4^3,3^4)$, $(5^2,4,3^4)$, (4^7), $(4^5,3^2)$, $(5,4^3,3^3)$, $(5^2,4^5)$, $(5,4^5,3)$, $(4^3,3^2,2^2)$, $(4^4,3^2,2)$, $(5,4^2,3^3,2)$, $(4^6,2)$, $(4^3,3^3,1)$;

$n=8$: (5^8), (4^8), $(5^2,4^6)$, $(6,4^7)$, $(4^4,3^4)$, $(5,4^2,3^5)$, $(4^6,3^2)$, $(5,4^6,3)$, $(4^3,3^4,2)$, $(4^7,2)$, $(4^4,3^3,1)$, $(5,4^2,3^4,1)$, $(4^3,3^3,2,1)$, $(4^6,3,1)$, $(5,4^6,1)$;

$n=9$: (4^9), $(4^3,3^5,1)$, $(4^8,2)$, $(4^7,3,1)$, $(5,4^7,1)$, $(4^3,3^4,1^2)$, $(4^7,1^2)$;

$n=10$: $(4^8,1^2)$.

Theorem 29 Let $n\geq 18$ and $\pi=(d_1,d_2,\cdots,d_n)\in NS_n$ be a positive graphic sequence with $d_6\geq 5$. Then π is potentially A_6-graphic if and only if $\pi_6\not\in\{(2),(2^2),(3,1),(3^2),(3,2,1),(3^2,2),(3^3,1),(3^2,1^2)\}$.

Yin and Chen [61] characterized the potentially $K_{r,s}$-graphic sequences for $r=2,s=3$ and $r=2,s=4$.

Theorem 30 Let $n\geq 5$ and $\pi=(d_1,d_2,\cdots,d_n)\in GS_n$. Then π is potentially $K_{2,3}$-graphic if and only if π satisfies the following conditions:

(1) $d_2\geq 3$ and $d_5\geq 2$;

(2)If $d_1=n-1$ and $d_2=3$, then $d_5=3$;

(3) $\pi\neq(3^2,2^4)$, $(3^2,2^5)$, $(4^3,2^3)$, $(n-1,3^5,1^{n-6})$ and $(n-1,3^6,1^{n-7})$.

Theorem 31 Let $n\geq 6$ and $\pi=(d_1,d_2,\cdots,d_n)\in GS_n$. Then π is potentially $K_{2,4}$-graphic if and only if π satisfies the following conditions:

(1) $d_2\geq 4$ and $d_6\geq 2$;

(2)If $d_1=n-1$ and $d_2=4$, then $d_3=4$ and $d_6\geq 3$;

(3) $\pi\neq(4^3,2^4)$, $(4^2,2^5)$, $(4^2,2^6)$, $(5^2,4,2^4)$, $(5^3,3,2^3)$, $(6,5^2,2^5)$, $(5^3,2^4,1)$, $(6^3,2^6)$, $(n-1,4^2,3^4,1^{n-7})$, $(n-1,4^2,3^5,1^{n-8})$, $(n-2,4^2,2^3,1^{n-6})$, and $(n-2,4^3,2^2,1^{n-6})$.

Chen [3] characterized the potentially K_5-2K_2-graphic sequences for $5\leq n\leq 8$. Hu and Lai [23] characterized the potentially K_5-P_3, K_5-A_3, K_5-K_3, $K_5-K_{1,3}$ and K_5-2K_2-graphic sequences where A_3 is $P_2\cup K_2$.

Theorem 32 Let $\pi=(d_1,d_2,\cdots,d_n)$ be a graphic sequence with $n\geq 5$. Then π is potentially K_5-P_3-graphic if and only if the following conditions hold:

(1) $d_1\geq 4$, $d_3\geq 3$ and $d_5\geq 2$.

(2) $\pi\neq(4,3^2,2^3)$, $(4,3^2,2^4)$ and $(4,3^6)$.

Theorem 33 Let $\pi=(d_1,d_2,\cdots,d_n)$ be a graphic sequence with $n\geq 5$. Then π is potentially K_5-A_3-graphic if and only if the following conditions hold:

(1) $d_4\geq 3$ and $d_5\geq 2$.

(2) $\pi\neq(n-1,3^3,2^{n-k},1^{k-4})$ where $n\geq 6$ and $k=4,5,\cdots,n-2$, n and k have the same parity.

(3) $\pi\neq(3^4,2^2),(3^6),(3^4,2^3),(3^6,2),(4,3^6),(3^7,1),(3^8),(n-1,3^5,1^{n-6})$ and $(n-1,3^6,1^{n-7})$.

Theorem 34 Let $\pi = (d_1, d_2, \cdots, d_n)$ be a graphic sequence with $n \geq 5$. Then π is potentially $K_5 - K_3$-graphic if and only if the following conditions hold:

(1) $d_2 \geq 4$ and $d_5 \geq 2$.

(2) $\pi \neq (4^2, 2^4)$, $(4^2, 2^5)$, $(4^3, 2^3)$ and (4^6).

Theorem 35 Let $\pi = (d_1, d_2, \cdots, d_n)$ be a graphic sequence with $n \geq 5$. Then π is potentially $K_5 - K_{1,3}$-graphic if and only if the following conditions hold:

(1) $d_1 \geq 4$ and $d_4 \geq 3$.

(2) $\pi \neq (4, 3^4, 2)$, (4^6), $(4^2, 3^4)$, $(4, 3^6)$, (4^7), $(4, 3^5, 1)$, $(n-1, 3^4, 1^{n-5})$ and $(n-1, 3^5, 1^{n-6})$.

Theorem 36 Let $\pi = (d_1, d_2, \cdots, d_n)$ be a graphic sequence with $n \geq 5$. Then π is potentially $K_5 - 2K_2$-graphic if and only if the following conditions hold:

(1) $d_1 \geq 4$ and $d_5 \geq 3$;

(2)

$$\pi \neq \begin{cases} (n-i, n-j, 3^{n-i-j-2k}, 2^{2k}, 1^{i+j-2}) \\ \ n-i-j \text{ is even;} \\ (n-i, n-j, 3^{n-i-j-2k-1}, 2^{2k+1}, 1^{i+j-2}) \\ n-i-j \text{ is odd.} \end{cases}$$

where $1 \leq j \leq n-5$ and $0 \leq k \leq [\frac{n-j-i-4}{2}]$.

(3) $\pi \neq (4^2, 3^4)$, $(4, 3^4, 2)$, $(5, 4, 3^5)$, $(5, 3^5, 2)$, (4^7), $(4^3, 3^4)$, $(4^2, 3^4, 2)$, $(4, 3^6)$, $(4, 3^5, 1)$,$(4, 3^4, 2^2)$, $(5, 3^7)$, $(5, 3^6, 1)$, (4^8), $(4^2, 3^6)$, $(4^2, 3^5, 1)$, $(4, 3^6, 2)$, $(4, 3^5, 2, 1)$, $(4, 3^7, 1)$, $(4, 3^6, 1^2)$, $(n-1, 3^5, 1^{n-6})$ and $(n-1, 3^6, 1^{n-7})$.

Hu and Lai [21] characterized the potentially $K_5 - C_4$-graphic sequences.

Theorem 37 Let $\pi = (d_1, d_2, \cdots, d_n)$ be a graphic sequence with $n \geq 5$. Then π is potentially $(K_5 - C_4)$-graphic if and only if the following conditions hold:

(1) $d_1 \geq 4$.

(2) $d_5 \geq 2$.

(3) $\pi \neq ((n-2)^2, 2^{n-2})$ for $n \geq 6$, where the symbol x^y stands for y consecutive terms x.

(4) $\pi \neq (n-k, k+i, 2^i, 1^{n-i-2})$ where $i = 3, 4, \cdots, n-2k$ and $k = 1, 2, \cdots, [\frac{n-1}{2}] - 1$.

(5) If $n = 6$, then $\pi \neq (4, 2^5)$.

(6) If $n = 7$, then $\pi \neq (4, 2^6)$.

Hu and Lai [22] characterized the potentially $K_5 - Z_4$-graphic sequences.

Theorem 38 Let $\pi = (d_1, d_2, \cdots, d_n)$ be a graphic sequence with $n \geq 5$. Then π is potentially $(K_5 - Z_4)$-graphic if and only if the following conditions hold:

(1) $d_1 \geq 4$, $d_2 \geq 3$ and $d_4 \geq 2$.

Hu, Lai and Wang[25] characterized the potentially K_5-P_4 and K_5-Y_4-graphic sequences where Y_4 is a tree on 5 vertices and 3 leaves.

Theorem 39 Let $\pi=(d_1,d_2,\cdots,d_n)$ be a graphic sequence with $n\geq 5$. Then π is potentially K_5-P_4-graphic if and only if the following conditions hold:

(1) $d_2\geq 3$.

(2) $d_5\geq 2$.

(3) $\pi\neq(n-1,k,2^t,1^{n-2-t})$ where $n\geq 5$, $k,t=3,4,\cdots,n-2$, and, k and t have different parities.

(4) For $n\geq 5$, $\pi\neq(n-k,k+i,2^i,1^{n-i-2})$ where $i=3,4,\cdots,n-2k$ and $k=1,2,\cdots,[\frac{n-1}{2}]-1$.

(5) If $n=6,7$, then $\pi\neq(3^2,2^{n-2})$.

Theorem 40 Let $\pi=(d_1,d_2,\cdots,d_n)$ be a graphic sequence with $n\geq 5$. Then π is potentially K_5-Y_4-graphic if and only if the following conditions hold:

(1) $d_3\geq 3$.

(2) $d_4\geq 2$.

(3) $\pi\neq(3^6)$.

Hu and Lai [24] characterized the potentially $K_{3,3}$ and K_6-C_6-graphic sequences.

Theorem 41 Let $\pi=(d_1,d_2,\cdots,d_n)$ be a graphic sequence with $n\geq 6$. Then π is potentially $K_{3,3}$-graphic if and only if the following conditions hold:

(1) $d_6\geq 3$;

(2) For $i=1,2$, $d_1=n-i$ implies $d_{4-i}\geq 4$;

(3) $d_2=n-1$ implies $d_3\geq 5$ or $d_6\geq 4$;

(4) $d_1+d_2=2n-i$ and $d_{n-i+3}=1(3\leq i\leq n-4)$ implies $d_3\geq 5$ or $d_6\geq 4$;

(5) $d_1+d_2=2n-i$ and $d_{n-i+4}=1(4\leq i\leq n-3)$ implies $d_3\geq 4$;

(6) $\pi=(d_1,d_2,3^4,2^t,1^{n-6-t})$ or $(d_1,d_2,4^2,3^2,2^t,1^{n-6-t})$ implies $d_1+d_2\leq n+t+2$;

(7) $\pi=(d_1,d_2,4,3^4,2^t,1^{n-7-t})$ implies $d_1+d_2\leq n+t+3$;

(8) For $t=5,6$, $\pi\neq(n-i,k+i,4^t,2^{k-t},1^{n-2-k})$ where $i=1,\cdots,[\frac{n-k}{2}]$ and $k=t,\cdots,n-2i$;

(9) $\pi\neq(5^4,3^2,2)$, (4^6), $(3^6,2)$, $(6^4,3^4)$, $(4^2,3^6)$, $(4,3^6,2)$, $(3^6,2^2)$, (3^8), $(3^7,1)$, $(4,3^8)$, $(4,3^7,1)$, $(3^8,2)$, $(3^7,2,1)$, $(3^9,1)$, $(3^8,1^2)$, $(n-1,4^2,3^4,1^{n-7})$, $(n-1,4^2,3^5,1^{n-8})$, $(n-1,5^3,3^3,1^{n-7})$, $(n-2,4,3^5,1^{n-7})$, $(n-2,4,3^6,1^{n-8})$, $(n-3,3^6,1^{n-7})$, $(n-3,3^7,1^{n-8})$.

Theorem 42 Let $\pi=(d_1,d_2,\cdots,d_n)$ be a graphic sequence with $n\geq 6$. Then π is potentially K_6-C_6-graphic if and only if the following conditions hold:

(1) $d_6\geq 3$;

(2) For $i=1,2$, $d_1=n-i$ implies $d_{4-i}\geq 4$;

(3) $d_2 = n-1$ implies $d_4 \geq 4$;
(4) $d_1 + d_2 = 2n-i$ and $d_{n-i+3} = 1(3 \leq i \leq n-4)$ implies $d_4 \geq 4$;
(5) $d_1 + d_2 = 2n-i$ and $d_{n-i+4} = 1(4 \leq i \leq n-3)$ implies $d_3 \geq 4$;
(6) $\pi = (d_1, d_2, d_3, 3^k, 2^t, 1^{n-3-k-t})$ implies $d_1+d_2+d_3 \leq n+2k+t+1$;
(7) $\pi = (d_1, d_2, 3^4, 2^t, 1^{n-6-t})$ implies $d_1 + d_2 \leq n+t+2$;
(8) $\pi \neq (n-i, k, t, 3^t, 2^{k-i-t-1}, 1^{n-2-k+i})$ where $i = 1, \cdots, [\frac{n-t-1}{2}]$ and $k = i+t+1, \cdots, n-i$ and $t = 4, 5, \cdots, k-i-1$;
(9) $\pi \neq (3^6, 2)$, $(4^2, 3^6)$, $(4, 3^6, 2)$, $(3^6, 2^2)$, (3^8), $(3^7, 1)$, $(4, 3^8)$, $(4, 3^7, 1)$, $(3^8, 2)$, $(3^7, 2, 1)$, $(3^9, 1)$, $(3^8, 1^2)$, $(n-1, 4^2, 3^4, 1^{n-7})$, $(n-1, 4^2, 3^5, 1^{n-8})$, $(n-2, 4, 3^5, 1^{n-7})$, $(n-2, 4, 3^6, 1^{n-8})$, $(n-3, 3^6, 1^{n-7})$, $(n-3, 3^7, 1^{n-8})$.

Xu and Hu[57] characterized the potentially $K_{1,4}+e$-graphic sequences. Chen and Li [8] characterized the potentially $K_{1,t}+e$-graphic sequences.

Theorem 43 Let $\pi = (d_1, d_2, \cdots, d_n)$ be a graphic sequence with $n \geq 5$. Then π is potentially $K_{1,4}+e$-graphic if and only if $d_1 \geq 4, d_3 \geq 2$.

Theorem 44 Let $t \geq 3$, $\pi = (d_1, d_2, \cdots, d_n)$ is a graphic sequence with $n \geq t+1$. Then π is potentially $K_{1,t}+e$-graphic if and only if $d_1 \geq t, d_3 \geq 2$.

Open Problems

Problem 1. Determine $\sigma(K_{r+1} - G, n)$ for G is a graph on k vertices and j edges which contains a graph $K_3 \bigcup K_{1,3}$ but not contains a cycle on 4 vertices and not contains Z_4, P_3.

Problem 2. Determine $\sigma(K_{r+1} - G, n)$ for $G = K_3 \bigcup iK_2 \bigcup jP_2 \bigcup tK_3$.

Problem 3. Determine $\sigma(K_{r+1} - G, n)$ for graph G which contains C_3, $C_4, \cdots, C_l$ but not contains a cycle on $l+1$ vertices$(4 \leq l \leq r)$.

Problem 4. Determine $\sigma(K_{r+1} - G, n)$ for graph G which contains C_3, $C_4, \cdots, C_{r+1}$.

Problem 5. Determine $\sigma(K_{r+1} - G, n)$ for small n.

Problem 6. Characterize potentially $K_{r+1} - G$-graphic sequences for the remaining G.

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